4D Variational Data Assimilation with Incompact3d using Large Eddy Models

Pranav Chandramouli

4D Variational Data Assimilation with Incompact3d using Large Eddy Models

Pranav CHANDRAMOULI
Supervised by: Etienne MEMIN & Dominique HEITZ

INRIA - Rennes (Fluminance)
Incompact3d Meeting - London, 2018

28/03/2018

Plan

1. Large Eddy Simulation
   - SGS Models
   - LES - Results
2. Data Assimilation
   - Introduction to Data Assimilation
   - Code Formulation
   - VDA with LES - Results

Objective

To Identify an apt Sub-Grid Scale (SGS) model for coarse LES in terms of statistical accuracy and numerical stability.

LES - SGS Models

Smagorinsky Variants :

$$\mathit{Classic} : \nu_t = (C_s * \Delta)^2 \bar{S}, \tag{1}$$

$$\mathit{Dynamic} : C_s^2 = -\frac{1}{2} \frac{\mathscr{L}_{ij} \bar{M}_{ij}}{M_{kl} \bar{M}_{kl}}. \tag{2}$$

where,

$$\mathscr{L}_{ij} = T_{ij} - \tilde{\tau}_{ij} \tag{3}$$

$$M_{ij} = \tilde{\bar{\Delta}}^2 |\tilde{\bar{S}}| \tilde{\bar{S}}_{ij} - \bar{\Delta}^2 (\widetilde{|\bar{S}| \bar{S}_{ij}}), \tag{4}$$

WALE :

$$\nu_t = (C_w \Delta)^2 \frac{(\varsigma_{ij}^d \varsigma_{ij}^d)^{3/2}}{(\bar{S}_{ij} \bar{S}_{ij})^{5/2} + (\varsigma_{ij}^d \varsigma_{ij}^d)^{5/4}} \tag{5}$$

LES - Models under Location Uncertainty (MULU)

$$\frac{\mathrm{d}\mathbf{X}_t}{\mathrm{d}t} = \boldsymbol{w}(\mathbf{X}_t, t) + \sigma(\mathbf{X}_t, t)\dot{\mathbf{B}} \tag{6}$$

- $\boldsymbol{w}(\mathbf{X}, t)$ is the large scale drift
- $\sigma(\mathbf{X}, t)d\dot{\mathbf{B}}_t$ stands for small scales represented by a noise

Mémin, Etienne. "Fluid flow dynamics under location uncertainty." Geophysical & Astrophysical Fluid Dynamics 108.2 (2014) : 119-146.

MULU

NS formulation as derived in Memin [2014] :

Mass conservation :

$$d_t\rho_t + \nabla\cdot(\rho\tilde{\boldsymbol{w}})dt + \nabla\rho\cdot\boldsymbol{\sigma}\,d\boldsymbol{B}_t = \frac{1}{2}\nabla\cdot(\boldsymbol{a}\nabla q)dt, \tag{7}$$

$$\tilde{\boldsymbol{w}} = \boldsymbol{w} - \frac{1}{2}\nabla\cdot\boldsymbol{a} \tag{8}$$

For an incompressible fluid :

$$\nabla\cdot(\boldsymbol{\sigma}\,d\boldsymbol{B}_t) = 0, \quad \nabla\cdot\tilde{\boldsymbol{w}} = 0, \tag{9}$$

Momentum conservation :

$$\left(\partial_t\boldsymbol{w} + \boldsymbol{w}\nabla^T(\boldsymbol{w} - \frac{1}{2}\nabla\cdot\boldsymbol{a}) - \frac{1}{2}\sum_{ij}\partial_{x_i}(a_{ij}\partial_{x_j}\boldsymbol{w})\right)\rho = \rho\boldsymbol{g} - \nabla p + \mu\Delta\boldsymbol{w}. \tag{10}$$

MULU

Modelling of $\boldsymbol{a}$:

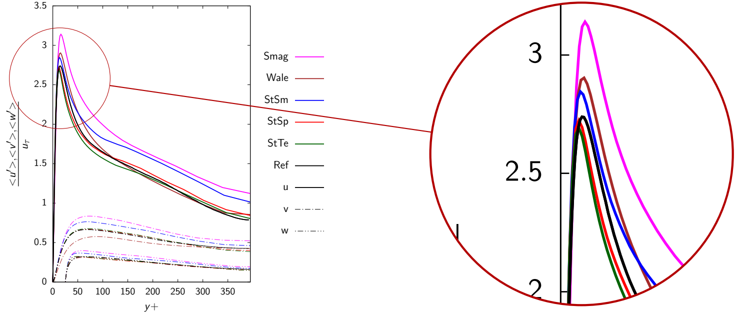
Stochastic Smagorinsky model (StSm) :

$$\boldsymbol{a}(\boldsymbol{x}, t) = C||\boldsymbol{S}||\mathbb{I}_3, \tag{11}$$

Local variance based models (StSp / StTe) :

$$\boldsymbol{a}(\boldsymbol{x}, n\delta t) = \frac{1}{|\Gamma| - 1} \sum_{x_i \in \eta(x)} (\boldsymbol{w}(x_i, n\delta t) - \bar{\boldsymbol{w}}(x, n\delta t))(\boldsymbol{w}(x_i, n\delta t) - \bar{\boldsymbol{w}}(x, n\delta t))^T C_{st}, \tag{12}$$

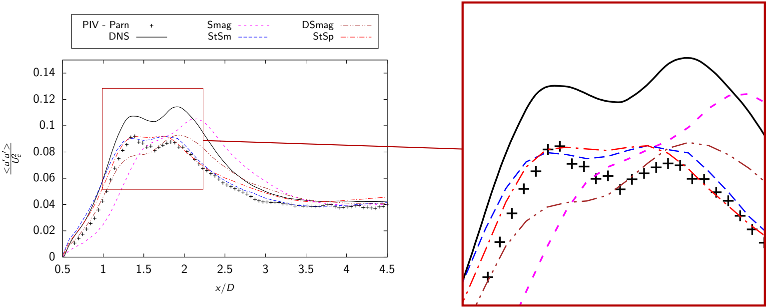
Channel Flow



Velocity fluctuation profiles for turbulent channel flow at $\mathrm{Re}_\tau = 395$

	$n_x \times n_y \times n_z$	$l_x \times l_y \times l_z$	Δx	Δy	Δz	Δt
cLES	48×81×48	6.28×2×3.14	0.13	0.005-0.12	0.065	0.002
Ref	256×257×256	6.28×2×3.14	0.024	0.0077	0.012	-

Wake Flow



$< u'u' >$ profile in the streamwise direction on the centre-line behind the cylinder.

	Re	$n_x \times n_y \times n_z$	$l_x/D \times l_y/D \times l_z/D$	$\Delta x/D$	$\Delta y/D$	$\Delta z/D$	$U\Delta t/D$
cLES	3900	241×241×48	20×20×3.14	0.083	0.024-0.289	0.065	0.003
PIV - Parn	3900	160×128×1	3.6×2.9×-	0.023	0.023	-	0.01
DNS	3900	1537×1025×96	20×20×3.14	0.013	0.0056-0.068	0.033	0.00075

Plan

Objective

An efficient method to incorporate Experimental Fluid Dynamics (EFD) as a guiding mechanism within Computational Fluid Dynamics (CFD) (Data Assimilation) - within the realm of Large Eddy Simulations (LES).

Data Assimilation - Types

- Sequential approach : Cuzol and Mémin (2009), Colburn et al. (2011), Kato and Obayashi (2013), Combes et al. (2015)
- **Variational approach (VDA)** : Papadakis and Mémin (2009), Suzuki et al. (2009), Heitz et al. (2010), Lemke and Sesterhenne (2013), Gronskis et al. (2013), Dovetta et al. (2014)
- Hybrid approach : Yang (2014)

VDA - General Formulation

Objective : Estimate the unknown true state of interest $\boldsymbol{x}^t(t, x)$

Formulation :

$$\partial_t \boldsymbol{x}(t, x) + \mathbb{M}(\boldsymbol{x}(t, x)) = \boldsymbol{q}(t, x), \tag{13}$$

$$\boldsymbol{x}(t_0, x) = \boldsymbol{x}_0^b + \boldsymbol{\eta}(x) \tag{14}$$

$$\boldsymbol{\mathcal{Y}}(t, x) = \mathbb{H}(\boldsymbol{x}(t, x)) + \epsilon(t, x) \tag{15}$$

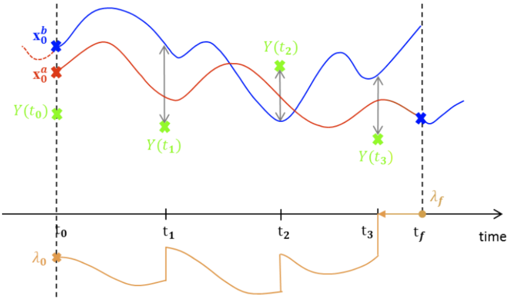
$\boldsymbol{q}(t, x)$ - model error (Covariance matrix $\mathbf{Q}$)
$\boldsymbol{\eta}(x)$ - background error (Covariance matrix $\mathbf{B}$)
$\epsilon(t, x)$ - observation error (Covariance matrix $\mathbf{R}$)

VDA - General Formulation

Cost Function :

$$
\begin{aligned}
J(\boldsymbol{x}_0) =& \frac{1}{2}(\boldsymbol{x}_0 - \boldsymbol{x}_0^b)^T \boldsymbol{B}^{-1}(\boldsymbol{x}_0 - \boldsymbol{x}_0^b) + \\
& \frac{1}{2}\int_{t_0}^{t_f} (\mathbb{H}(\boldsymbol{x}_t) - \mathcal{Y}(t))^T \boldsymbol{R}^{-1}(\mathbb{H}(\boldsymbol{x}_t) - \mathcal{Y}(t)).
\end{aligned}
\tag{16}
$$

4DVar - Adjoint Method

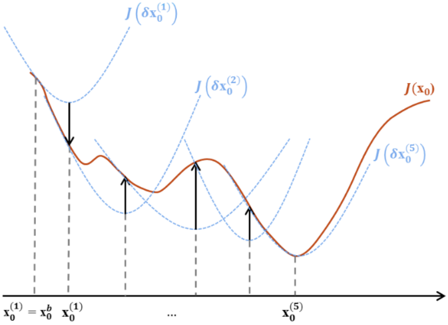


Evolution of the state of interest (x_0) as a function of time

$$\frac{\partial J}{\partial \eta} = -\lambda(\mathbf{t_o}) + \mathbf{B}^{-1}(\partial \mathbf{x}(\mathbf{t_0}) - \partial \mathbf{x_0}). \tag{17}$$

Le Dimet, Francois-Xavier, and Olivier Talagrand. "Variational algorithms for analysis and assimilation of meteorological observations : theoretical aspects." Tellus A : Dynamic Meteorology and Oceanography 38.2 (1986) : 97-110.

4DVar - Incremental Optimisation



Evolution of the cost function ($J(x_0)$) as a function of time

Courtier, P., J. N. Thépaut, and Anthony Hollingsworth. "A strategy for operational implementation of 4D-Var, using an incremental approach." Quarterly Journal of the Royal Meteorological Society 120.519 (1994) : 1367-1387.

Additional Control

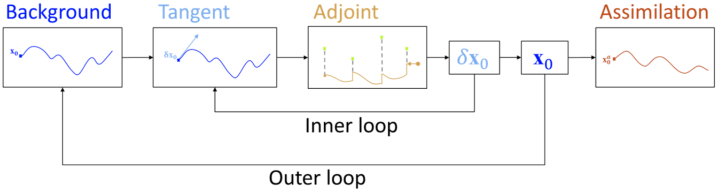
Minimisation of J with respect to additional incremental control parameter δu : $\delta\gamma^{(i)} = \{\delta\mathbf{x_0^{(i)}}, \delta\mathbf{u^{(i)}}\}$

$$J(\delta\gamma^{(i)}) = \frac{1}{2}||\delta\mathbf{x_0^{(i)}} + \mathbf{x_0^{(i)}} - \mathbf{x_0^{(b)}}||^{\mathbf{2}}_{\mathbf{B}^{-1}} + \frac{1}{2}\int_{t_0}^{t_f} ||\delta\mathbf{u^{(i)}} + \mathbf{u^{(i)}} - \mathbf{u^{(b)}}||^{\mathbf{2}}_{\mathbf{B_c}} + \ldots \tag{18}$$

constrained by :

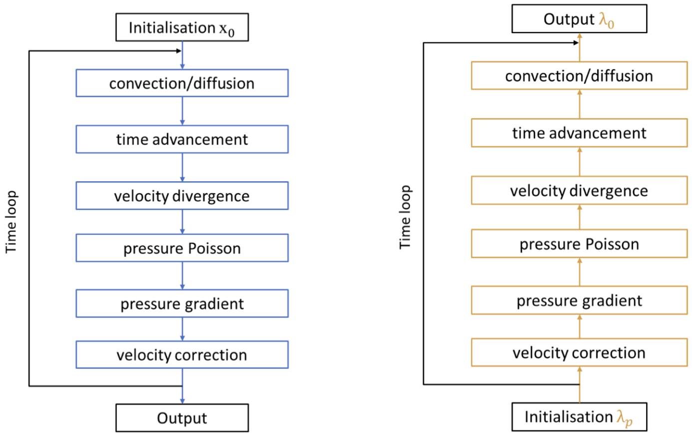
$$\partial_t\delta\mathbf{x^{(i)}} + \partial_{\mathbf{x}}\mathbb{M}(\mathbf{x^{(i)}})\cdot\partial\mathbf{x^{(i)}} + \partial_{\mathbf{u}}\mathbb{M}(\mathbf{u^{(i)}})\cdot\partial\mathbf{u^{(i)}} = \mathbf{0} \tag{19}$$

4DVar - Flow Chart

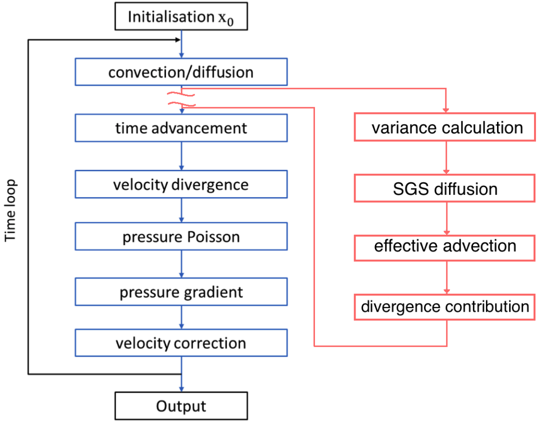


4DVar incremental Data Assimilation using Adjoint methodology.

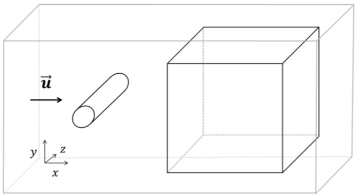
Incompact3d (Laizet et al. 2010) - DNS



Incompact3d (Laizet et al. 2010) - LES



Synthetic Assimilation at Re 3900

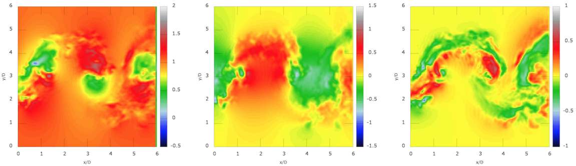


Optimisation Parameter - $\mathbf{U}(\mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{t_0})$
Control Parameters - $\mathbf{U_{in}}(\mathbf{1}, \mathbf{y}, \mathbf{z}, \mathbf{t})$
$C_{st}(x, y, z)$

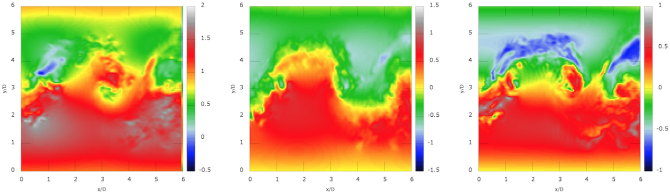
	Re	$n_x \times n_y \times n_z$	$l_x/D \times l_y/D \times l_z/D$	$U\Delta t/D$	Duration
Full Domain	3900	$361 \times 361^s \times 48$	$20 \times 20 \times 3.14$	0.003	$40100\Delta t$
Observation	3900	$145^i \times 145^i \times 48$	$6 \times 6 \times 3.14$	0.003	$100\Delta t$
4DVAR	3900	$145 \times 145 \times 48$	$6 \times 6 \times 3.14$	0.003	$100\Delta t$

s - Stretched ; i - Interpolated

Background Velocity - Biased

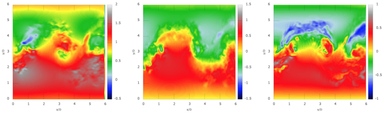


$+ \sin y^3$ noise

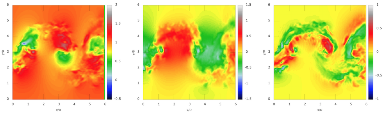


Assimilated Velocity

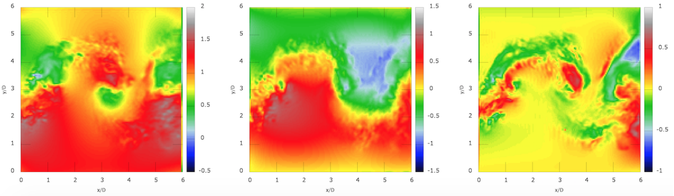
Background



Reference

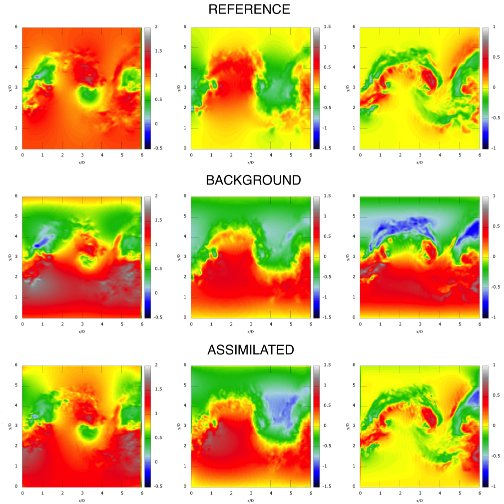


4DVAR Algorithm

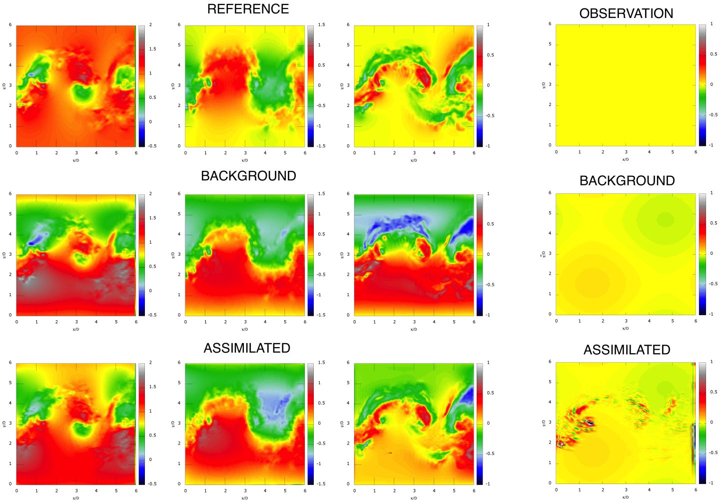


Assimilated

Collated Results - Initial Condition Correction



LES Coefficient Estimation - Preliminary Results



Conclusions

- Efficient cost reduction is possible using LES with VDA
- An improvement on the background is obtained using LES based VDA
- An estimation of the LES coefficient can be obtained using VDA

Future Work

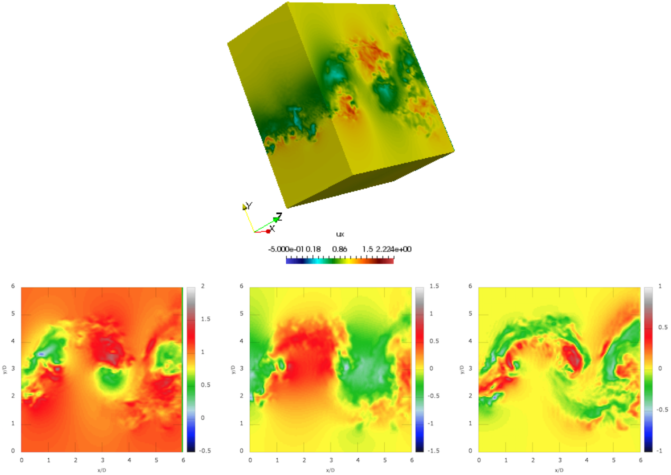
- Perform LES DA with experimental data
- Use sliding windows in VDA
- Include outlet condition in optimisation algorithm
- Assimilate over a larger temporal domain

References

- P. Chandramouli, E. Memin, D. Heitz, and S. Laizet. Coarse large-eddy simulations in a transitional wake flow with flow models under location uncertainty. Comput. Fluids, 168 :170-189, 2018.
- P. Chandramouli, E. Memin, D. Heitz, and S. Laizet. A Comparative Study of LES Models Under Location Uncertainty. In Proceedings of the Congrés Français de Mécanique., 2017b.
- P. Chandramouli, E. Memin, and D. Heitz. 4d Turbulent Wake Reconstruction using Large Eddy Simulation based Variational Data Assimilation. Delft, Netherlands, 2017a.
- E. Memin. Fluid flow dynamics under location uncertainty. Geophys. Astro. Fluid, 108(2) : 119-146, March 2014.
- A. Gronskis, D. Heitz, and E. Memin. Inflow and initial conditions for direct numerical simulation based on adjoint data assimilation. J. Comput. Phys., 242 :480-497, June 2013.

Thanks for your attention

Reference - Synthetic Velocity Fields



Models under Location Uncertainty

Modelling of $\boldsymbol{a}$:

Stochastic Smagorinsky model (StSm) :

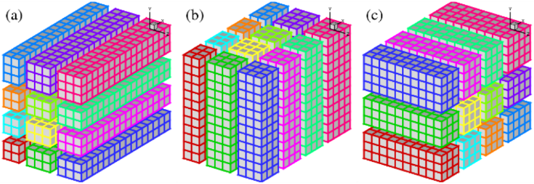
$$\boldsymbol{a}(\boldsymbol{x}, t) = C||\boldsymbol{S}||\mathbb{I}_3, \tag{20}$$

Local variance based models (StSp / StTe) :

$$\boldsymbol{a}(\boldsymbol{x}, n\delta t) = \frac{1}{|\Gamma| - 1} \sum_{x_i \in \eta(x)} (\boldsymbol{w}(x_i, n\delta t) - \bar{w}(x, n\delta t))(\boldsymbol{w}(x_i, n\delta t) - \bar{w}(x, n\delta t))^T C_{sp}, \tag{21}$$

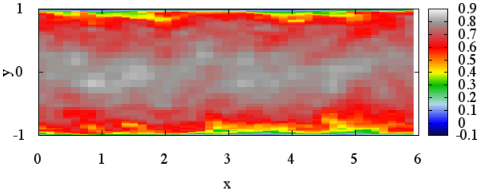
Plan

Flow Solver - Incompact3d



- Fortran based parallelised flow solver - 2Decomp & FFT (Laizet S. & Lamballais E. (2009))
- Simulates the incompressible Navier Stokes equations
- Spectral pressure treatment with collocated or staggered grid formulation

Flow Specifications

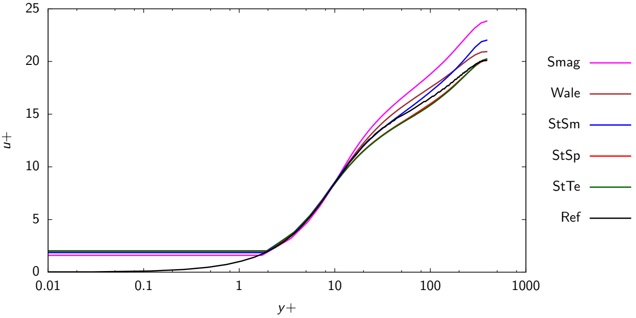


Instantaneous streamwise velocity contour

	$n_x \times n_y \times n_z$	$l_x \times l_y \times l_z$	Δx	Δy	Δz	Δt
cLES	48×81×48	6.28×2×3.14	0.13	0.005-0.12	0.065	0.002
Ref	256×257×256	6.28×2×3.14	0.024	0.0077	0.012	-

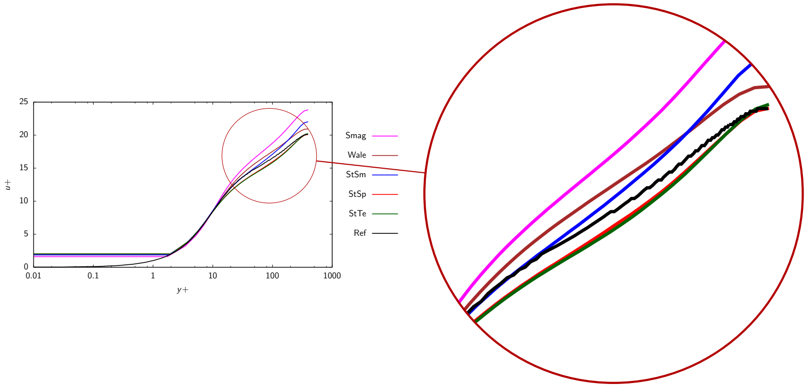
Channel Flow Statistics

Mean Velocity Profile :

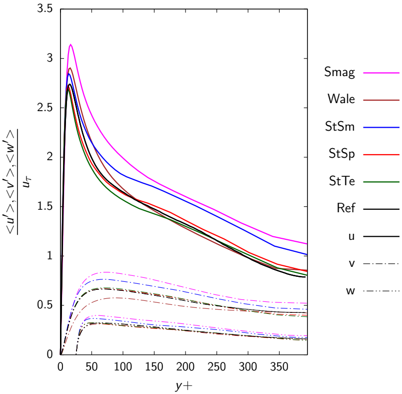


Mean velocity profile for turbulent channel flow at $\mathrm{Re}_\tau = 395$

Channel Flow

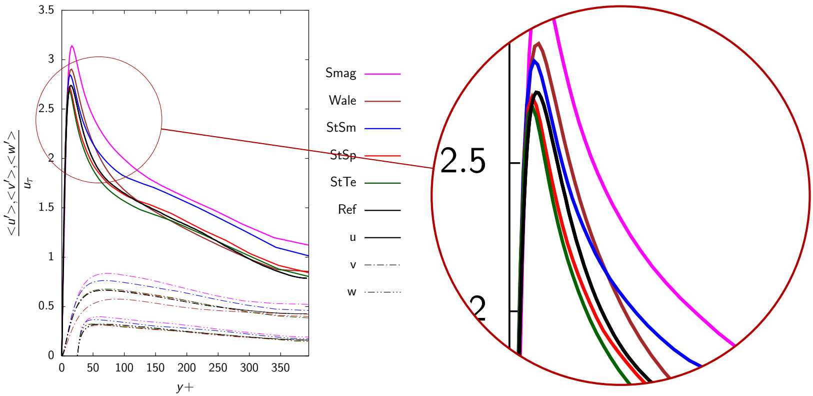


Channel Flow

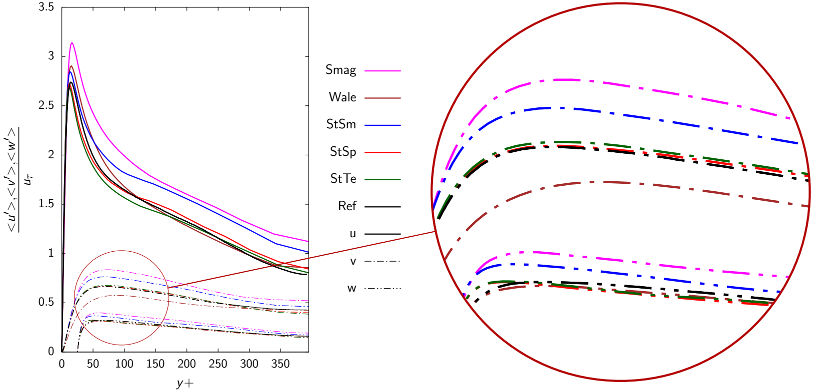


Velocity fluctuation profiles for turbulent channel flow at $\mathrm{Re}_\tau = 395$

Channel Flow



Channel Flow

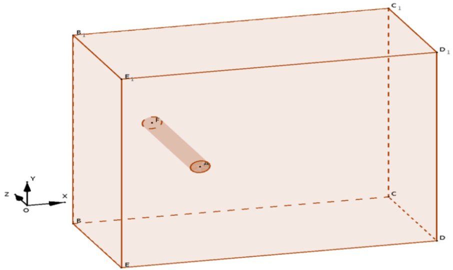


Plan

3 Channel Flow

4 Wake Flow

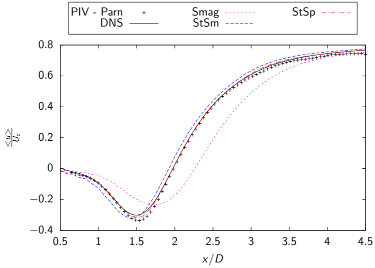
Flow Parameters



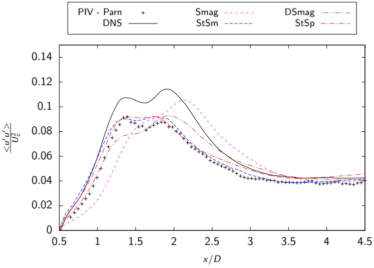
	Re	$n_x \times n_y \times n_z$	$l_x/D \times l_y/D \times l_z/D$	$\Delta x/D$	$\Delta y/D$	$\Delta z/D$	$U\Delta t/D$
cLES	3900	241×241×48	20×20×3.14	0.083	0.024-0.289	0.065	0.003
PIV - Parn	3900	160×128×1	3.6×2.9×-	0.023	0.023	-	0.01
LES - Parn	3900	961×961×48	20×20×3.14	0.021	0.021	0.065	0.003
DNS	3900	1537×1025×96	20×20×3.14	0.013	0.0056-0.068	0.033	0.00075

Flow Statistics

Centerline statistical comparison :

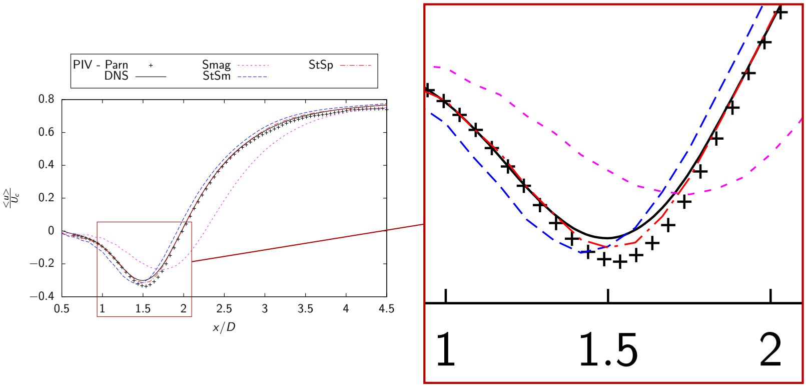


Mean streamwise velocity in the streamwise direction along the centerline behind the cylinder.

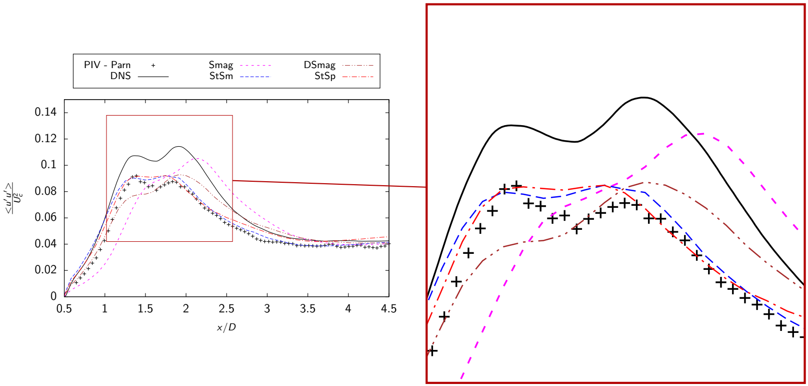


Fluctuating streamwise velocity in the streamwise direction along the centerline behind the cylinder.

Flow Statistics

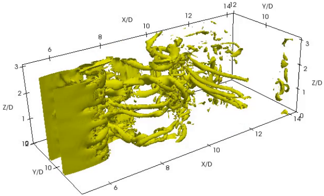


Flow Statistics

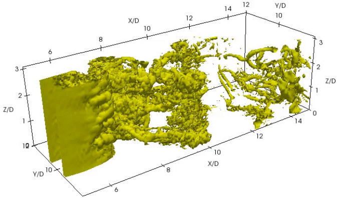


Vorticity Structures

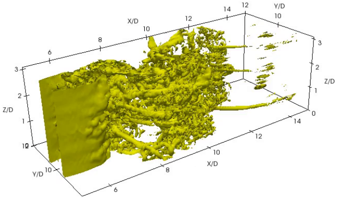
Smagorinsky



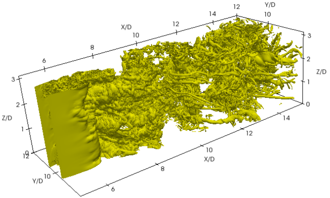
Stochastic Smagorinsky



Stochastic Spatial

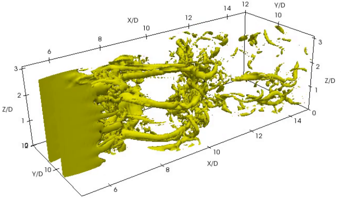


DNS

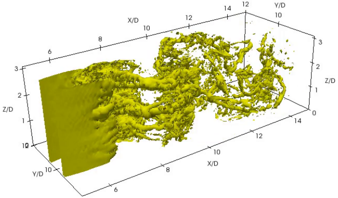


Vorticity Structures

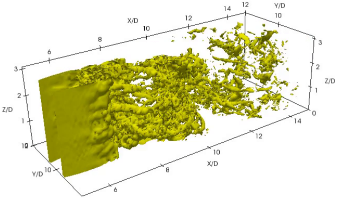
Smagorinsky



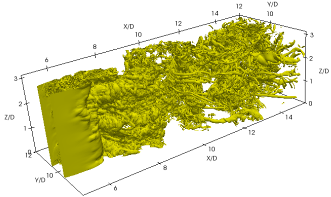
Stochastic Smagorinsky



Stochastic Spatial



DNS



Simulation Cost

Model	w/o model	Smag	StSm	StSp
Computational Cost [s/iteration]	1.34	2.877	3.654	5.711

TABLE – Computational cost [s] per iteration for each model.

Cost of StSp under coarse resolution = **0.46%** cost of DNS

4D-Var with Incompact3d - DNS

Algorithm 27 `Incompact3d_b` $(\mathbf{u}^0,\bar{\mathbf{u}}^0)$

Initialization: $k = N,\mathbf{u}^0,\bar{\mathbf{u}}^N = 0,\ \bar{F}^{N-1} = 0,\ \bar{F}^{N-2} = 0$

while k>0 **do**

  `corgp_b`$(\mathbf{u}^k,\bar{\mathbf{u}}^k,\nabla p,\nabla \bar{p})$

  `gradp_b`$(\nabla p,\nabla \bar{p},p,\bar{p})$

  `poisson_b`$(p,\bar{p},\mathbf{u}^{**},\bar{\mathbf{u}}^{**})$

  `pre_correc_b`$(p,\bar{p})$

  `intt_b`$(\mathbf{u}^*,\bar{\mathbf{u}}^*,F^{k-1},\bar{F}^{k-1},F^{k-2},\bar{F}^{k-2})$

  `convdiff_b`$(\mathbf{u}^{k-1},\bar{\mathbf{u}}^{k-1},F^{k-1},\bar{F}^{k-1},F^{k-2},\bar{F}^{k-2})$

  $k = k - 1$

end while